

# WEYL'S PRINCIPLE, COSMIC TIME AND QUANTUM FUNDAMENTALISM<sup>†</sup>

S.E. RUGH\* AND H. ZINKERNAGEL\*\*

## Abstract

We examine the necessary physical underpinnings for setting up the cosmological standard model with a global cosmic time parameter. In particular, we discuss the role of Weyl's principle which asserts that cosmic matter moves according to certain regularity requirements. After a brief historical introduction to Weyl's principle we argue that although the principle is often not explicitly mentioned in modern standard texts on cosmology, it is implicitly assumed and is, in fact, necessary for a physically well-defined notion of cosmic time. We finally point out that Weyl's principle might be in conflict with the wide-spread idea that the universe at some very early stage can be described exclusively in terms of quantum theory.

## 1 Introduction

A basic characteristic of the Friedmann-Lemaître-Robertson-Walker (FLRW) model is its  $t$  parameter which is employed by cosmologists to trace back the evolution of the universe to its early stages. In a previous examination, we defended a 'time-clock' relation which asserts that time, in order to have a physical basis, must be understood in relation to physical processes which act as 'cores' of clocks (Rugh and Zinkernagel 2009). In particular, we argued that a necessary physical condition for *interpreting* the  $t$  parameter of the FLRW model as cosmic time in some 'epoch' of the universe is the (at least possible) existence of a physical process which can function as a core of a clock in the 'epoch' in question.<sup>1</sup>

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\*Symposium, 'The Socrates Spirit', Section for Philosophy and the Foundations of Physics, Hellebækgade 27, Copenhagen N, Denmark (*e-mail: rugh@symposion.dk*)

\*\*Department of Philosophy I, Granada University, 18071 Granada, Spain (*e-mail: zink@ugr.es*).

<sup>1</sup>One of our results was that there are interesting problems for making this  $t \leftrightarrow$  time interpretation, and thus establishing a physical basis for cosmic time (in particular for a cosmic time *scale*), at least at  $\sim 10^{-11}$  seconds after the "big bang" – that is, approximately 30 orders of magnitude before (in a backwards extrapolation of the FLRW model) Planck scales are reached.

In this paper we shall argue, in conformity with – but independently of – the time-clock relation, that the very set-up of the standard (FLRW) model in cosmology with a global time is closely linked to the motion (and properties) of cosmic matter.<sup>2</sup> It is often assumed that the FLRW model may be derived just from the *cosmological principle* which states that the universe is spatially homogeneous and isotropic (on large scales). It is much less well known that another assumption, often called Weyl’s principle, is necessary – or, at least, have been claimed to be necessary – in order to arrive at the FLRW model and, in particular, its cosmic time parameter. In a version close to Robertson’s (1933) (we shall discuss various formulations later), the principle states:

*Weyl’s principle:* The world lines of galaxies, or ‘fundamental particles’, form (on average) a spacetime-filling family of non-intersecting geodesics converging towards the past.

The importance of Weyl’s principle is that it provides a reference frame based on an expanding ‘substratum’ of ‘fundamental particles’. In particular, if the geodesic world lines are required to be orthogonal to a series of space-like hypersurfaces, a comoving reference frame is defined in which constant spatial coordinates are “carried by” the fundamental particles. The time coordinate is a cosmic time which labels the series of hypersurfaces, and which may be taken as the proper time along any of the particle world lines.

Insofar as the Weyl principle is necessary for the notion of cosmic time in the FLRW model, it clearly becomes important to examine whether the properties and motion of matter are compatible with the Weyl principle as we go back in cosmic time.<sup>3</sup> If a point is reached at which this is not the case, then it appears not to be physically justified to contemplate ‘earlier’ epochs. Doing so would involve extrapolating the FLRW model into a domain where the fundamental assumptions (needed to build up the model) are no longer valid and the model would lose its physical basis (see also the discussion in Rugh and Zinkernagel 2009, p. 5).

In the following, we first briefly review the early history of Weyl’s principle and question a claim, found in some of the recent literature on this principle, to the effect that the principle has been replaced by the cosmological principle. We then show that although the Weyl principle is not often mentioned explicitly in modern texts on cosmology, it is nevertheless in these texts in implicit form (and, we argue, necessarily so). We finally discuss and question the prospect of satisfying Weyl’s principle, and hence define cosmic time, at a very ‘early phase’ of the universe, if this phase is thought to be describable exclusively in terms of quantum theory.

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<sup>2</sup>Whereas our aforementioned study examines the physical basis for time both locally and globally we shall assume in the present manuscript that spacetime is physically well-defined locally.

<sup>3</sup>A related question may of course be made concerning the cosmological principle, see e.g. Weinberg (1973, p. 407).

## 2 A very brief history of Weyl’s principle

The early history and reception of Weyl’s principle – sometimes denoted postulate, assumption or hypothesis – have been chronicled e.g. by North 1990, and more recently by Bergia and Mazzoni (1999) and Goenner (2001), see also Ehlers (2009). In this section we sketch a few important points about the historical development, and we argue against an apparent consensus among Bergia and Mazzoni (1999) and Goenner (2001) which takes Weyl’s principle to have been rendered redundant by the cosmological principle.

Weyl first introduced his principle in 1923, with the appearance of the 5th and revised version of his *Raum, Zeit, Materie*, in connection with a discussion of de Sitter’s solution to Einstein’s field equations. In Weyl’s 1926 formulation of his principle (which he here called hypothesis), it reads:<sup>4</sup>

...the world lines of the stars [in later contexts, galaxies] form a sheaf [bundle], which rises in a given direction from the infinitely distant past, and spreads out over the hyperboloid [representing de Sitter’s model] in the direction of the future, getting broader and broader. [Quoted from Goenner (2001, p. 121), our inserts]

This principle, or hypothesis, amounts to specifying a choice of congruence (a family of non-crossing curves which fills spacetime) of timelike geodesics to represent the world lines of the cosmological substratum, see e.g. Goenner (2001, p. 120). As Weyl emphasized, such an assumption concerning the choice of congruence is necessary to derive an unambiguous cosmological redshift in de Sitter’s model; see also Bergia and Mazzoni (1999, pp. 336-338). As for the possible empirical support of his principle, Weyl mentions (6 years before Hubble’s 1929 paper) that “it appears that the velocities between distant celestial objects on average increase with their mutual separations” (quoted from Ehlers 2009, p. 1655).

Weyl (1923, p. 1664) notes that on his hypothesis the stars (galaxies) belong “to the same causally connected system with a common origin”.<sup>5</sup> Moreover, Weyl hints (same page) that this causal connectedness of the stars implies an assumption of the ‘state of rest’ of the stars which is “the only one compatible with the homogeneity of space and time”. On this point, Goenner (2001, p. 120) comments that Weyl thus “indicates that his hypothesis implies the existence of a common time parameter or, turned around, that the stars have a common instantaneous rest system”. While the implication of a cosmic time is consistent with Robertson’s use of Weyl’s principle (see below), Goenner also notes (p. 126) that Weyl never explains in detail why his chosen congruence also implies a common rest frame for the galaxies (and thus a cosmic time).

As we shall see in section 3, the issue of whether or not a common time (a common rest frame) is implied by Weyl’s principle may be responsible for the difference

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<sup>4</sup>Weyl’s formulations in his 1923 writings are more convoluted, see Goenner (2001) and Bergia and Mazzoni (1999).

<sup>5</sup>According to Ehlers (2009, p. 1657) this “implies that each particle can be influenced by all others at any time; in modern parlance there is no particle horizon”.

in formulations of this principle in the literature. While Robertson is somewhat ambiguous concerning what is included in the Weyl principle (see section 3), he notes (1933, p. 65) that the reintroduction in cosmology of a significant simultaneity (a cosmic time) implied by Weyl’s postulate is permissible since observations support the idea that galaxies (on average) are moving away from each other with a mean motion which represents the actual motion to within relatively small and unsystematic deviations.<sup>6</sup>

Whereas, for Weyl, the selection of a particular congruence of curves as world lines to represent cosmic matter was originally merely a specific property of the de Sitter universe, it later became (e.g. for Robertson) a necessary assumption for constructing cosmological models. This role of Weyl’s principle is emphasized by Ehlers in Bertotti (1990, p. 29); see also Weyl (1930, p. 937):

H. Weyl in 1923 points out that to have a cosmological model one *has to specify*, besides a space-time (M,g), a congruence of timelike curves to represent the mean motion of matter. [Our emphasis]

Now, Weyl introduced his principle in de Sitter space where (unlike the FLRW model) there is no unique choice of congruence (this is why de Sitter’s cosmos can be written either as a static or an expanding universe), see e.g. Ellis (1990, p. 100). But, as we shall discuss further in section 3 and 4, even if the choice of congruence is unique in the FLRW model, it is still crucial that the actual matter content of the universe is (on average) well represented by this congruence.

Bergia and Mazzoni (1999, p. 339) note: “In his 1929 paper, Robertson had given no justification for his introduction of a cosmic time. As we have just seen, he did offer some in 1933, guided by Weyl’s principle. Therefore the continuity between Weyl’s and the cosmological principle seems fairly well established”. This quote might indicate that the cosmological principle somehow replaced the Weyl principle but such an idea would, in our assessment, be misleading both for historical and conceptual reasons.<sup>7</sup> For Robertson (1933, p. 65) clearly states that he uses two (in fact four) assumptions amounting to *both* Weyl’s principle and the cosmological principle.<sup>8</sup> Furthermore, Robertson notes that the cosmic time implied by Weyl’s principle “allows us to give a relatively precise formulation of the assumption that our ideal approximation to the actual world is spatially uniform” (1933, p. 65), and he thus suggests that the Weyl principle is actually a *precondition* for the cosmological principle (we shall pursue this theme further in section 3).

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<sup>6</sup>Robertson’s empirical justification for the introduction of a cosmic time stands in contrast to Friedmann’s statement (1922, p. 1993): “In the expression for  $ds^2$ ,  $g_{14}$ ,  $g_{24}$ ,  $g_{34}$  can be made to vanish by corresponding choice of the time coordinate, or, shortly said, time is orthogonal to space. It seems to me that no physical or philosophical reasons can be given for this second assumption; it serves exclusively to simplify the calculations”.

<sup>7</sup>The term “cosmological principle” is due to Milne in 1933, though it can already implicitly be found in Einstein’s 1917 paper, see e.g. North (1990, p. 157).

<sup>8</sup>The four assumptions, schematically, are: (1) a congruence of geodesics; (2) hypersurface orthogonality; (3) homogeneity; and (4) isotropy. In section 3 we return to the relation between (1), (2) and Weyl’s principle in Robertson (1933).

In a spirit which seems similar to that of Bergia and Mazzoni, Goenner (2001, p. 126) notes in connection with the literature of the 1940s: “In fact, Weyl’s hypothesis had become superfluous and was replaced by the *cosmological principle*, i.e. the hypothesis that, in the space sections, no point and no direction are preferred”. Apparently, Goenner’s assessment of the (ir-)relevance of Weyl’s principle for today’s cosmology is similar: “Weyl’s stature in mathematics and science may ... explain why the hypothesis still is mentioned in some modern books on gravitation and cosmology, notably by authors not specialized in cosmological research” (2001, p. 127). This statement fits well with the fact that Weyl’s principle is often absent in an explicit form in current cosmology textbooks. However, as we shall argue in the following section, Weyl’s principle is, at least implicitly, still present (and necessarily so) in the main texts on cosmology.

### 3 Weyl’s principle in standard texts on cosmology

In some cosmology textbooks, e.g. Bondi (1960), Raychaudhuri (1979) and Narlikar (2002), the importance of Weyl’s principle is emphasized, and explicitly referred to, when the physical basis of the comoving frame, cosmic time, and the FLRW model are outlined. For instance, the derivation of the FLRW metric in Narlikar (2002, p. 107 ff.) is explicitly built on two assumptions, namely:

1. Weyl’s postulate (Narlikar): The world lines of galaxies [or ‘fundamental particles’] form a bundle of non-intersecting geodesics orthogonal to a series of spacelike hypersurfaces.
2. The cosmological principle: The universe, on large scales, is spatially homogeneous and spatially isotropic.<sup>9</sup>

In Narlikar’s formulation of Weyl’s postulate (which includes the orthogonality criterion; see below), this postulate is sufficient to build up a comoving reference frame in which the constituents of the universe are at rest (on average) relative to the comoving coordinates: The trajectories,  $x_i = \text{constant}$ , of the constituents are freely falling geodesics, and the requirement that the geodesics be orthogonal to the spacelike hypersurfaces translates into the requirement  $g_{0i} = 0$ , which (globally) resolves the space-time into space and time (a 3+1 split). We have  $g_{00} = 1$  if we choose the time coordinate  $t$  so that it corresponds to proper time ( $dt = ds/c$ ) along the lines of constant  $x_i$ , i.e.  $t$  corresponds to clock time for a standard clock at rest in the comoving coordinate system.<sup>10</sup> The metric can thus be written in

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<sup>9</sup>We note that while Weyl’s principle and the cosmological principle allow for the possibility to set up the FLRW model with a *global* cosmic time, the implementation of these principles can only be motivated physically if we already have a physical foundation for the concepts of ‘space’ and ‘time’ *locally* – otherwise we cannot apply concepts like “spacelike”, “spatially homogeneous”, “spatially isotropic”, which appear in the definitions of these principles.

<sup>10</sup>That the world lines are geodesics implies that  $g_{00}$  depends only on  $x_0$ , and so that  $g_{00}$  can be set to unity by a suitable coordinate transformation, see e.g. Narlikar (2002, p. 109).

*synchronous* form in which the spacelike hypersurfaces are surfaces of simultaneity for the comoving observers (see also e.g. MTW 1973, p. 717):

$$ds^2 = c^2 dt^2 - g_{ij} dx^i dx^j \quad i, j = 1, 2, 3. \quad (1)$$

In Narlikar (2002), the role of the cosmological principle is then to simplify the spatial part of this metric in order to get the standard FLRW form:<sup>11</sup>

$$ds^2 = c^2 dt^2 - R^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right\} \quad (2)$$

Narlikar's discussion of the assumptions going into the derivation of the FLRW line element seems to follow Robertson (1933) closely with one notable difference: Narlikar takes orthogonality of the matter world lines to the series of space-like hypersurfaces as *being part of* Weyl's principle (and hence, implicitly, that the existence of a comoving frame follows directly from this principle). By contrast, Robertson states Weyl's principle as in section 1 (i.e. as the principle that matter is represented by a congruence of diverging geodesics) and add, as a *further* assumption, that the space-like hypersurfaces are orthogonal to the congruence of geodesics.<sup>12</sup> In any case, whether or not the assumption of hypersurface orthogonality is included in Weyl's principle, it is clear that one can impose the requirement of the congruence being orthogonal to the hypersurfaces only given that there *is* a congruence.<sup>13</sup>

Given the importance of Weyl's principle in Robertson (1933), it may at first be surprising that other (than the above mentioned) standard text books on general relativity and modern cosmology, such as Misner, Thorne and Wheeler (MTW) (1973), Wald (1984), Peebles (1993) or Hawking and Ellis (1973), have no explicit reference to Weyl's principle. As far as we can see, however, the reason is simply that the Weyl principle is assumed implicitly in these books at an early stage of setting up the FLRW model. To see this, consider first Ellis' (1990, p. 99) clarification (cf. also the quote of Ehlers in section 2 above):

It is important to realise that a cosmological model is specified only when a 4-velocity  $u^a$  representing the average motion of matter in the universe

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<sup>11</sup>Although there is a preferred choice of the congruence of world lines (a preferred reference frame) in the FLRW model (see below), there are many different coordinate representations of this model; see e.g. Krasinski (1997, p. 11 and p. 14 - 16) who outline at least five different coordinate representations.

<sup>12</sup>However, Robertson is actually somewhat ambiguous about what exactly is included in, and implied by, Weyl's principle. For, right after introducing *both* assumptions (congruence and hypersurface orthogonality), he mentions (1933, p. 65): "The possibility of thus introducing in a natural and significant way this *cosmic time*  $t$  we consider as guaranteed by Weyl's postulate, which is in turn a permissible extrapolation from the astronomical observations". Perhaps this ambiguity is related to Weyl's own insufficient explanation, mentioned in section 2, of whether a comoving frame ('state of rest' of the stars) follows directly from his principle.

<sup>13</sup>One can have a congruence which is not orthogonal to a series of spacelike hypersurfaces but not, of course, a hypersurface orthogonal congruence which is not a congruence! Note however the underlying coupled problem: The specification of a congruence (is it hypersurface orthogonal?, are the world lines geodesics?, etc.) depends on  $g_{\mu\nu}$ , and the specific form of  $g_{\mu\nu}$  depends in turn on the choice of congruence (the reference frame).

has been specified as well as the space-time metric  $g_{\mu\nu}$ ; the observable relations in the model are determined by the choice of this 4-velocity, or equivalently of the associated fundamental world-lines.

As mentioned in section 2, the preferred choice of a 4-velocity or, equivalently, a congruence of world lines, to represent the average motion of matter is unique in the FLRW case.<sup>14</sup> But still, the congruence plays a fundamental role since the symmetry constraints of homogeneity and isotropy are imposed *with respect to* such a congruence, cf. e.g. Ellis (1999):

We start by assuming large-scale spatial homogeneity and isotropy *about a particular family of worldlines*. The RW models used to describe the large-scale structure of the universe embody those symmetries exactly in their geometry. It follows that comoving coordinates can be chosen ....  
[Our emphasis]

Another way of stating this point is that isotropy can be satisfied only in a particular reference frame or for a particular class of fundamental observers (other observers moving with respect to these will not see isotropy). Indeed, such a fundamental class of observers (congruence) is part of the definition of isotropy, cf. e.g. Wald (1984, p. 93) (see also MTW 1973, p. 714):

A spacetime is said to be (spatially) *isotropic* at each point if there exists a congruence of timelike curves (i.e. observers), with tangents denoted  $u^a$ , filling the spacetime ... [such that] ... it is impossible to construct a geometrically preferred tangent vector orthogonal to  $u^a$ .

Thus, Weyl's principle – in the general sense of matter being well represented by a congruence of world lines – is a precondition for the cosmological principle; the former can be satisfied without the latter being satisfied but not vice versa.

As hinted above in connection with Robertson, the specification of a congruence of world lines representing matter is a necessary but not sufficient condition for setting up a cosmic time.<sup>15</sup> Only with the additional requirement of the congruence of world lines being hypersurface orthogonal do we get a sufficient condition. In terms of the 4-velocity field, a sufficient condition for having cosmic time is that

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<sup>14</sup>The mentioned equivalence follows if precisely one non-zero 4-velocity vector is assigned to every point of the manifold (the world line is obtained as the integral curve of  $u^a$ ). Ehlers (1961) introduces a three-dimensional family of timelike curves to represent the world lines of the matter elements with respect to an arbitrary local coordinate system,  $x^a = x^a(y^\alpha, s)$ . He then derives the 4-velocity of the matter elements by differentiating the coordinates  $x^a$  (i.e.  $u^a = \partial x^a / \partial s$ ) with respect to the proper time  $s$  along the world lines. Subsequently the acceleration  $\dot{u}$  of the matter elements can be defined, and this acceleration vanishes ( $\dot{u} = 0$ ) if the world lines move along geodesics.

<sup>15</sup>For instance, Gödel's expanding and rotating solution from 1952 satisfies, as far as we can see, Weyl's principle in Weyl's 1926 (and Robertson's 1933) formulation. But this solution does not have a cosmic time (at least not for high rates of rotation, see e.g. Belot (2005, p. 27)). See also Ellis (1996) for a detailed discussion both of Gödel's static and expanding rotating solutions of Einstein's field equations.

the motion of matter is (on average) described by this field and that the motion is irrotational (corresponding to the 4-velocity having zero vorticity; see e.g. Ellis (1996)).<sup>16</sup> Asserting the existence of the 4-velocity field (representing matter) is of course prior to inquiring about its vorticity. This is just another way of repeating that the condition that matter can be well described by Weyl’s principle is necessary for having cosmic time.

To require that the motion of matter is well represented by a congruence of world lines (i.e. to impose Weyl’s principle in the general sense) is to require that the matter world lines are *non-crossing* (of course, this can only be true on average, see below). This non-crossing of world lines is built into the construction of the comoving frame with respect to which cosmic time is defined. As described e.g. in MTW (1973, p. 715 ff), see also Wald (1984, p. 95) and Weinberg (1972, p. 338), an arbitrary grid of space coordinates  $(x^1, x^2, x^3)$  (constant labels) are laid out on a spacelike hypersurface (of homogeneity). These coordinates are “propagated off” and throughout all spacetime *by means of* the world lines of the cosmological fluid with proper (= cosmic) time measured along any of the fluid world lines (so the coordinates are “carried by” the world lines). Since the world lines of the cosmological fluid are used to propagate the coordinates it is crucial that there is *no crossing* of the world lines (i.e. that the family of world lines constitutes a congruence), as we would otherwise have the same spacetime point described by different (incompatible) coordinates.<sup>17</sup> As Narlikar puts it (2002, p. 108):

It is worth emphasizing the importance of the non-intersecting nature of world lines. If two galaxy world lines did intersect, our [comoving] coordinate system above would break down, for we would then have two different values of  $x^\mu$  specifying the same point in spacetime (the point of intersection).

How can Weyl’s principle be fulfilled in the *real* universe? Typical ordinary velocities of (nearby) galaxies relative to each other are  $\langle v \rangle \sim 1/1000 \times c$  (MTW 1973, p. 711) and, indeed, some galaxies do collide. Likewise with the more fundamental constituents in earlier phases of the universe. Thus the fundamental world lines in

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<sup>16</sup>The 4-velocity field may be decomposed into rotation (vorticity), shear and expansion components, see e.g. Ehlers (1961, p. 1228) or MTW (1973, §22.3). As concerns the connection between zero vorticity and hypersurface orthogonality, Malament (2006, p. 251) presents a nice picture: “Think about an ordinary rope. In its natural twisted state, the rope cannot be sliced by an infinite family of slices in such a way that each slice is orthogonal to all fibers. But if the rope is first untwisted, such a slicing is possible. Thus orthogonal sliceability is equivalent to fiber untwistedness. The proposition extends this intuitive equivalence to the four-dimensional ‘spacetime ropes’ (i.e. congruences of worldlines) encountered in relativity theory. It asserts that a congruence is irrotational (i.e. exhibits no twistedness) iff it is, at least locally, hypersurface orthogonal.”

<sup>17</sup>Note that in Minkowski spacetime there is no unique congruence of world lines (no unique preferred frame), no (preferred) cosmic time, and no need to impose the non-crossing criterion – but also that one does not need the world lines of the material constituents to propagate (set up) the coordinates. It is however possible to do so, and in that case it *is* necessary that the world lines are non-crossing, see e.g. Peebles (1993, p. 250).



the Weyl principle must be some ‘average world lines’ associated with the average motion of the fundamental particles (in order to “smooth out” any crossings).<sup>18</sup>

At present and for most of cosmic history, the comoving frame of reference can be identified as the frame in which the cosmic microwave background radiation looks isotropic (see e.g. Peebles 1993, p. 152), and cosmic matter is (above the homogeneity scale) assumed to be described as dust particles with zero pressure which fulfill Weyl’s principle. In the early radiation phase, matter is highly relativistic (moving with random velocities close to  $c$ ), and the Weyl principle is not satisfied for a typical particle but one may still introduce fictitious averaging volumes in order to create substitutes for ‘galaxies which are at rest’; see e.g. Narlikar (2002, p. 131).

However, above the electroweak phase transition (before  $10^{-11}$  seconds ‘after’ the big bang), all constituents are massless and move with velocity  $c$  in any reference frame. There will thus be no constituents which are comoving (at rest).<sup>19</sup> One might attempt to construct mathematical points (comoving with a reference frame) like the above mentioned center of mass (or, in special relativity, center of energy) out of the massless, ultrarelativistic gas particles, but this procedure requires that length scales be available in order to e.g. specify how far the particles are apart (which is needed as input in the mathematical expression for the center of energy). As discussed in Rugh and Zinkernagel (2009) the only option for specifying such length scales (above the electroweak phase transition) will be to appeal to speculative physics, and the prospects of satisfying Weyl’s principle (and have a cosmic time) will therefore also rely on speculations beyond current well-established physics.

We conclude that it is instrumental that some averaging procedure is made in order to yield a non-crossing family of world lines (a congruence). Whether this is possible when matter is described by quantum theory (e.g. in the very early universe) is the question we address in the next section.

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<sup>18</sup>A closely related problem is to average out inhomogeneities in the matter distribution (such averaging procedures have been developed to a large degree of sophistication, see e.g. Krasinski (1997, pp. 263 - 275)). It is a highly non-trivial problem, and it was emphasized already by Gödel (1949, p. 560), that the necessary averaging over large volumes will introduce an arbitrariness in the definition of cosmic time depending on the details of the averaging process and the size of the regions considered (see also North (1990, p. 360) and Dieks (2005, p. 11 - 12)).

<sup>19</sup>This conclusion may also be reached by noting that the set-up of the FLRW model requires matter (the energy-momentum tensor) to be in the form of a perfect fluid, as this is the only form compatible with the FLRW symmetries, see e.g. Weinberg (1972, p. 414). And a source consisting of pure radiation is not sufficient since one cannot effectively simulate a perfect fluid by “averaging over pure radiation”: Krasinski (1997, p. 5 - 9) notes that the energy-momentum tensor in cosmological models may contain many different contributions, e.g. a perfect fluid, a null-fluid, a scalar field, and an electromagnetic field. But he also emphasizes that whereas a scalar field source is compatible with the FLRW geometry (since it acts as a stiff perfect fluid with equation of state  $p = \rho$ ), a source of pure null fluid or pure electromagnetic field is not compatible with the FLRW geometry, and solutions with such energy-momentum sources have no FLRW limit (see Krasinski 1997, p. 13).

## 4 Cosmic time with quantum matter?

We have seen that Weyl’s principle cannot be disregarded in the FLRW model as it is either implicitly or explicitly included among the fundamental principles used to set up this model. The question therefore arises: What could be candidates for the “Weyl substratum” which, at epochs when no galaxies are present, can form substitutes for (on average) non-intersecting galaxies at rest?

The empirical adequacy of both Weyl’s principle and the cosmological principle depends on the actual arrangement and motion of the physical constituents of the universe. As we go backwards in time it may become increasingly difficult to satisfy these physical principles since, as mentioned in section 3, the nature of the physical constituents is changing from galaxies, to relativistic gas particles, and to entirely massless particles moving with velocity  $c$ . In particular, the Weyl principle refers to a non-crossing family of (fluid or particle) world lines, that is, to classical or classicalized particle-like behavior of the material constituents. This makes it difficult even to formulate the Weyl principle (let alone decide whether it is satisfied) if some period in cosmic history is reached where the ‘fundamental particles’ are to be described by wave-functions  $\psi(x, t)$  referring to (entangled) quantum constituents. What is a ‘world line’ or a ‘particle trajectory’ then? Unless one can specify a clear meaning of ‘non-intersecting trajectories’ in a contemplated quantum ‘epoch’ it would seem that the very notion of cosmic time, and hence the notion of ‘very early universe’ is compromised.

This last problem of identifying a Weyl substratum within a quantum description arises most clearly on a “quantum fundamentalist” view according to which the material constituents of the universe could be described *exclusively* in terms of quantum theory at some early stage of the universe.<sup>20</sup> On such a quantum fundamentalist view, the following question naturally arises

*The cosmic measurement problem:* If the universe, either its content or in its entirety, was once (and still is) quantum, how can there be (apparently) classical structures now?

We call this the “cosmic measurement problem” since it addresses the standard quantum measurement problem in the cosmological context. While many aspects of the cosmic measurement problem have been addressed in the literature, the perspective which we would like to add is that the problem is closely related to providing a physical basis for the (classical) FLRW model with a (classical) cosmic time parameter.<sup>21</sup>

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<sup>20</sup>For instance, Kiefer notes that “The Universe was essentially ‘quantum’ at the onset of inflation” (Joos et al. 2003, p. 208).

<sup>21</sup>Note the temporal aspect of the cosmic measurement problem: Not only are classical structures less fundamental since they are derivable from quantum structures, but they are also *temporally* secondary to the original quantum state of the universe. Depending on the cosmic epoch of interest, various levels of the cosmic measurement problem can be distinguished, for instance (see e.g. Kiefer and Joos 1999): (1) How to get a classical spacetime out of quantum spacetime? (2) How to get classical structures from quantum fields (in a classical spacetime background) — for instance in

Our point is that if cosmic time in the FLRW model is crucially dependent on a (prior) classical or classicalized behaviour of the material constituents of the universe, then one can hardly (assume a quantum fundamentalist view and) approach the cosmic measurement problem by asserting a gradual emergence of classicality framed *in terms of* a cosmic time.

An often attempted response to the cosmic measurement problem is to proceed via the idea of decoherence. Within such an approach one may imagine that quantum particles in the early universe (like particles in a bubble chamber) will move along ‘tracks’ (instead of being wave functions spread out in space) — due to the interaction of the quantum constituents with the environment (that is, the environment of all the other particles are constantly ‘monitoring’ the particle wave function in question). However, there are reasons to question whether decoherence has sufficient explanatory power for the quantum fundamentalist (e.g. whether decoherence is sufficient to explain the building up of a Weyl substratum). First, as is widely known, decoherence cannot by itself solve the measurement problem and explain the emergence of the classical world (see e.g. Landsman 2006). Furthermore, as already indicated, if decoherence is to provide the classical structures (in the cosmological context), it cannot — as is usually assumed in environmental induced decoherence — be a process in (cosmic) time, insofar as classical structures (non-crossing world lines) are needed from the start to define cosmic time. Finally, a general worry about decoherence has been expressed e.g. by Anastopoulos (2002): “...a sufficiently classical behaviour for the environment seems to be necessary if it is to act as a decohering agent and we can ask what has brought the environment into such a state ad-infinitum”.<sup>22</sup>

Due to these limitations of the decoherence idea in the present context, the quantum fundamentalist is (in our view) still faced with the question of whether a comoving Weyl substratum can be constructed from (non classicalized) quantum constituents (wave functions). Apart from (but related to) the mentioned concerns about decoherence in this context, one may ask what ‘moves’ according to the quantum description? ¿From the point of view of a Born interpretation, the wave function in quantum theory is not a real wave but rather a probabilistic object. The evolution of a wave function  $\psi(\vec{x}, t)$  therefore appears insufficient to provide a physical basis for the fluid particles comprising the Weyl substratum since in the quantum description (in the Born interpretation) no physical object moves from a definite place  $\mathcal{A}$  to another place  $\mathcal{B}$ . Only a mathematical entity  $\psi(\vec{x}, t)$  — the symbolic representation of the quantum system — ‘moves’.<sup>23</sup>

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an early inflationary universe? (3) How to get a measurement apparatus to show definite results (the standard measurement problem)?

<sup>22</sup>A further problem is that while the split between system and environmental degrees of freedom may be natural in earth-based experimental arrangements, it appears less obvious in the context of the early universe. Thus, while Kiefer and Joos (1999) appear to assume that various subsets of constituents in the universe can successively classicalize one another via decoherence (starting with the gravitational degrees of freedom), Anastopoulos (2002) points out that the environment/system splitting seems to be arbitrary in the context of general relativity.

<sup>23</sup>In our assessment, also the local space and time concepts require a physical foundation in

It is not obvious that this problem will be more tractable if instead of one particle we have quantum systems composed of many constituents (see also Landsman 2006, p. 492). The early universe is envisaged to be described by a collection of interacting quantum fields. In general, these (matter and radiation) fields will be in an entangled state in which it is far from clear that individual particle trajectories are discernible. Thus, even with many constituents it is still not clear that something actually moves from one place to another. As a consequence, there may not be a well-defined notion of particle trajectories (let alone non-crossing particle trajectories) in which case no Weyl substratum can be identified. In that situation, no cosmic time can be defined and it thus seems difficult to maintain the quantum fundamentalist view of an early quantum ‘epoch’ of the universe.

As a mathematical study, the FLRW model may be extrapolated back arbitrarily close to  $t = 0$ . But as a physical model nobody believes it ‘before’ the Planck time. As we have argued, however, there are interesting problems with establishing a physical basis for the FLRW model with a cosmic time, even before (in a backward extrapolation from now) we might reach an ‘epoch’ in which theories of quantum gravity may come into play.

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terms of the material constituents (cf. the time-clock relation in Rugh and Zinkernagel 2009). In the quantum context (quantum mechanics as well as quantum field theory) we are therefore faced with an interesting circularity: The wavefunction  $\psi = \psi(\vec{x}, t)$  is defined on classical spacetime  $(\vec{x}, t)$  but spacetime has in turn to be constructed with reference to the material building blocks, that is, to the wave functions  $\psi$  themselves.

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